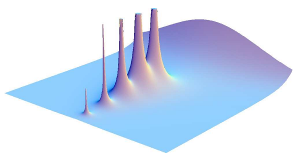

Physics 170

Exercise D — Fourier Transforms

Due 28 February 2005

Frequently one's data is contaminated with noise, and this noise may be large compared to the signal. One manner of proceeding is to modulate the signal at a fixed frequency and look for the power at the modulation frequency in the data. In this exercise you will use a fast Fourier transform routine to compute the power spectrum of a time series of Gaussian noise and of that same noise after adding a small sinusoidal signal.

Gaussian Noise

Gaussian (or "white") noise of unit amplitude is a zero-average signal (*e.g.*, voltage) with unit standard deviation that follows a Gaussian probability distribution. The built-in random number generator in many programming environments generates pseudo-random numbers that are **uniformly** distributed on the unit interval [0,1) (not distributed according to a Gaussian distribution). Some numerical packages (*e.g.*, MATLAB and Igor, among others) have a built-in function producing Gaussian noise. Others have only a random function that generates uniform random numbers.

The following Java code shows how to use the uniformly distributed random numbers from such a random function to generate Gaussian noise. See *Numerical Recipes* for details on the mathematical justification for why this routine gives Gaussian deviates.

```
/** gaussDeviate() returns an array of 2 doubles, each of which
    is a Gaussian deviate. See section 7.2 in Numerical Recipes.
*/
double [] gaussDeviate() {
    double v1, v2, r;
    do {
        v1 = 2.0 * Math.random() - 1.0;
        v2 = 2.0 * Math.random() - 1.0;
        r = v1 * v1 + v2 * v2;
    } while ( r > 1 );
    double fac = Math.sqrt( -2.0 * Math.log(r) / r );
    double [] res = new double( v1 * fac, v2 * fac );
    return res;
}
```

1. Write or use a program to prepare a time series of white noise having at least 512 data points. Compute its power spectrum using the FFT in an environment of your choosing.

2. Assemble a time series consisting of white noise plus a sinusoid of amplitude α. Compute its power spectrum for a variety of values of α. In your judgment, what is the minimum value of α for which the sinusoid is distinctly seen in the spectrum? Does it depend on the number of data points?